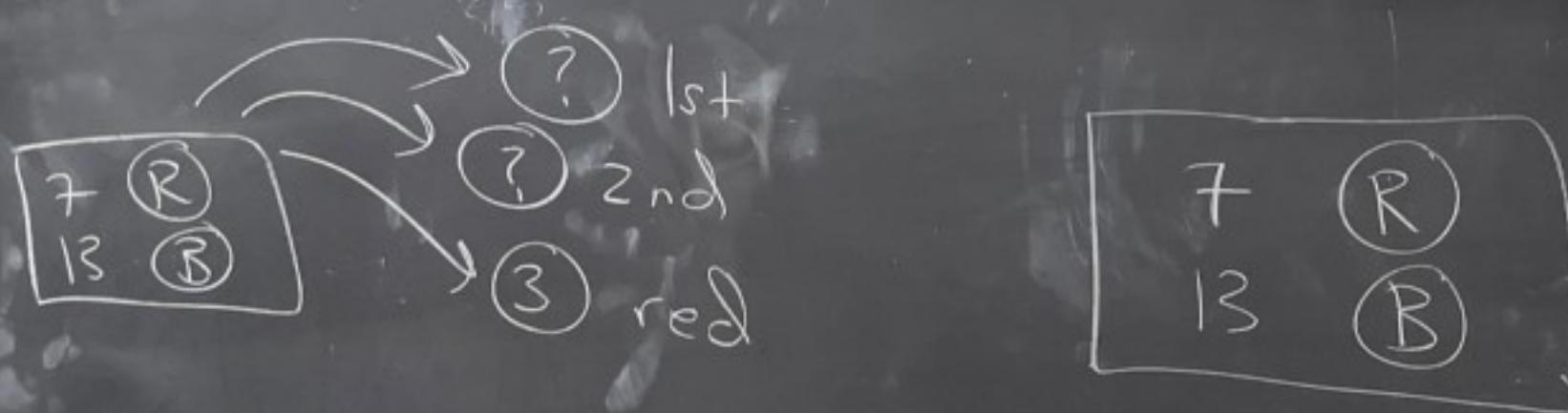


HW 3

(12)

Box with 7 red and 13 blue balls.



Take 2 balls out of  
the box and discard  
them without looking.

Then draw a 3rd ball  
and you notice its red.

What's the prob. the two  
discarded balls are blue?

Let  $BB$ ,  $BR$ ,  $RR$  be the events that  
the first two balls where blue/blue,  
blue/red, or red/red.

Let  $R$  be the event the 3rd ball is red.

We want  $P(BB|R)$

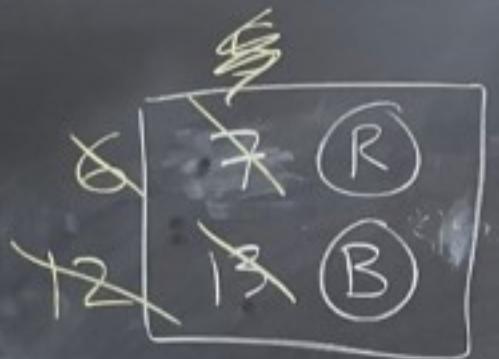
$$P(BB|R) = \frac{P(BB \cap R)}{P(R)} = \frac{P(R|BB) \cdot P(BB)}{P(R)}$$

$\uparrow$

$P(BB \cap R) = P(R|BB) \cdot P(BB)$

$P(R|BB) = \frac{P(R \cap BB)}{P(BB)} = \frac{P(BB \cap R)}{P(BB)}$

7  
11



$$P(BB) = \frac{\binom{13}{2}}{\binom{20}{2}} = \frac{13 \cdot 12}{20 \cdot 19} = \boxed{\frac{78}{190}}$$

$$\binom{13}{2} = \frac{13!}{2! 11!} = \frac{13 \cdot 12 \cdot 11!}{2! 11!} \\ = \frac{13 \cdot 12}{2}$$

$$\binom{n}{2} = \frac{n(n-1)}{2} \quad \star$$

$$P(R|BB) = \frac{\binom{7}{1}}{\binom{18}{1}} = \boxed{\frac{7}{18}}$$

after 2 blues  
taken out have



$$\frac{P(R|BB) \cdot P(BB)}{P(R)} = P(BB|R)$$

$\textcircled{R} \textcircled{R}$     $\frac{\binom{5}{1}}{\binom{18}{1}} = \frac{5}{18} \textcircled{R}$

$\textcircled{B}$     $\frac{13}{18}$

$\frac{21}{190} \cdot \frac{5}{18}$

$\frac{\binom{7}{2}}{\binom{20}{2}} = \frac{21}{190}$

$\textcircled{R} \textcircled{B}$     $\frac{\binom{6}{1}}{\binom{18}{1}} = \frac{6}{18} \textcircled{B}$

$\textcircled{B}$     $\frac{12}{18}$

$\frac{91}{190} \cdot \frac{6}{18}$

$\frac{\binom{13}{2}}{\binom{20}{2}} = \frac{78}{190}$

$\textcircled{B} \textcircled{B}$     $\frac{7}{18} \textcircled{R}$

$\textcircled{B}$     $\frac{11}{18} \textcircled{B}$

$\frac{78}{190} \cdot \frac{7}{18}$

First 2  
balls

Third  
ball

$$P(R) = \frac{21}{190} \cdot \frac{5}{18} + \frac{91}{190} \cdot \frac{6}{18} + \frac{78}{190} \cdot \frac{7}{18}$$

$$= \frac{7}{20}$$

Answer

$$P(BB|R) = \frac{P(R|BB) \cdot P(BB)}{P(R)} = \frac{(\frac{7}{18})(\frac{78}{190})}{\frac{7}{20}}$$

$$= \boxed{\frac{26}{57}} \approx \boxed{0.456}$$